Visual Multiplexing

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Outline

- Motivation
- Multiplexing in communication
- Categories of visual multiplexing
- Evidence: How did it work in visualization?
- Evidence: Do humans have such abilities?
- Evidence: Does it mathematically make sense?
- Conclusion
Initial Motivation

- In 2012-2013, we were working on summarizing video data in *Cardiovascular Magnetic Resonance* (CMR) Imaging.
- Many different data fields were derived from the video data.
# Temporal Multi-field Data

## Table 2: Different fields and attributes associated with CMR imagery data.

<table>
<thead>
<tr>
<th>Field/Attribute</th>
<th>Data Type (per point)</th>
<th>Symbol</th>
<th>Layer</th>
<th>Visual Mapping</th>
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<td><strong>Primary Fields</strong></td>
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Related Work

- Overlaying Methodologies
  - Map overlay
  - Multi-field visualization
  - Comparative visualization

- Cognitive sciences

- Information theory and data communication

http://learnpracticalgis.com/how-to-overlay-maps/
Multiplexing in Communication

- **Frequency Division (FDM)**

- **Time Division (TDM)**

- **Space Division (SDM)**

- **Code Division (CDM)**

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**References**

- http://www.cse.iitk.ac.in/users/dheeraj/cs425/lec04.html
Process: Literature Studies

- Identify various solutions for multivariate, multi-layer and multi-field visualization.
- Make small cards for many successful examples of such visualization.
Process: Categorization

- Brainstorm meetings to study different ways for grouping different example cards.
- Many categorization schemes were proposed.
- Gradually evolve to a scheme with 10 categories.
Type A: Partition a Space
Type B: Partition a Time Period

- Image 1: Cloud symbol indicating rain or stormy weather.
- Image 2: Green box with "5°C" indicating temperature.
- Image 3: Blue circle with "9" possibly indicating another measure or number.
Multiplexing in Visualization

Location $p$ can be associated with $X$ in the source data or determined by a spatial mapping.

Perceived information may include estimated values and relationships with data conveyed by other signals.

Data objects

$X = \langle x_1, x_2, \ldots, x_k \rangle$ at $p$
Multiplexing in Visualization

Let \( X = \langle x_1, x_2, \ldots, x_k \rangle \) at \( p \).

The vis-encoder processes \( X \) and sends it through the vis-link (consisting of many vis-channels).

At the receiver, the vis-decoder recovers the information about \( X \) at \( p \).

The spatial domain \( D \) and the temporal domain \( T \) are involved in the process, along with other signals and noise.
Type C: Introduce Partial Occlusion
Type D: Use a ‘Hollow’ Visual Channel
Type E: Introduce Translucent Occlusion
Type F: Use an Integrated Visual Channel
Type G: Depict a Continuous Field
Type H: Shift a Visual Channel
Type I: Use Periodic Motion
Type J: Assume A Priori Knowledge (1)
Visual multiplexing is an important phenomenon in visualization. It can be utilized to create effective visual designs.
Type J: Visual Language (3)
How did it work in visualization?

(a) Robertson et al. [RFF*08]
Type B

(b) Everts et al. [EBRI09]
Type C

(c) Bair, House [BH07]
Types C, D, G

(d) Treavett, Chen [TC00]
Type D, J

(e) Collins et al. [CPC09]
Types C, E

(f) Guo et al. [GXY12]
Type E

(g) Kindlmann, Westin [KW06]
Type F

(h) Ware [War09]
Types G, J
How did it work in visualization?

(i) Chen et al. [CPL*11]
Types C, G, J

(j) Saito et al. [SMY*05]
Type C, H

(k) Drocourt et al. [DBS*11]
Type H

(l) Ware, Plumlee [WP13]
Type I

(m) Viola et al. [VFSG06]
Type E, J

(n) Correa et al. [CSC06]
Type H, J

(o) Botchen et al. [BBS*08]
Types E, G, H, J

(p) Maguire et al. [MRSS*12]
Type J
Do Humans Have such Abilities?

- **Human Vision System**
  - 130M retinal receptors
  - optical nerve: 1.2M axons
  - fovea resolution: 10K points
  - rapid eye movement

- **Memory**
  - Sensory memory
  - Short-term (working) memory
  - Long-term memory

- **Gestalt principles**

- **Other cognitive abilities**
  - Learning, Reasoning, ...

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http://www.ecse.rpi.edu/~schubert/Light-Emitting-Diodes-dot-orgchap16/F16-01%20Human%20eye.jpg

http://www.human-memory.net/types.html
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  - Type C: Partial occlusion
  - Type D: ‘Hollow’ visual channel
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  - Type J: Knowledge
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**Types of Visual Categorization**

- **Type A:** Partition a space
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Does it Mathematically Make Sense?

- A puzzle in [Chen & Jänicke, 2010]

Diagram:

- Data Space
- Visualization Space
- Display Space

- Data Space Entropy: $H$
- Visualization Capacity (Visualization Space Entropy): $V(G)$
- Display Space Capacity: $D$

$V(G) << D <= 1$
An Illustrative Example

DSU = 0.0093

\[ DSU = 2 \times 0.0093 - \varepsilon \]

Probability of no collision or minor collision (at 4 or fewer location) = 0.997
Making Use of Underutilized Display Space Capacity

DSU = 0.93%

DSU ≈ 1.83%

DSU ≈ 3.72%

DSU ≈ 7.44%

DSU ≈ 14%
How about When Every Pixel is Used?

Data Space

Visualization Space

Display Space

Data Space Entropy $H$

Visualization Capacity (Visualization Space Entropy) $V(G)$

Display Space Capacity $D$

$V(G) \ll 1$
Conclusions

Observation:
- A common phenomenon in visualization.
- Humans can decode many types of visual multiplexing effortlessly.
- Support comparative and multivariate visualization.
- An intrinsic way to access underutilized display bandwidth.
- A priori knowledge can help.

Uses of this categorization:
- Assist visualization designers in exploring different visual encoding schemes systematically.
- Recognize and utilize users’ knowledge.
- Explore opportunities to create new knowledge.